Volumes of Solids of Revolution: Disk/Washer and Shell Methods
Sandra Peterson, LearningLab

For problems 1 - 2, let R be the region bounded by the given curves. Sketch R. If R is revolved about the x-axis, find the volume of the solid of revolution (a) by the disk/washer method, and (b) by the shell method. Show that the results are the same.

1. \( y = x^2 \)
   \( y = 2x \)

2. \( y = \sqrt{x} \)
   \( y = x^3 \)

For problems 3 - 4, let R be the region bounded by the given curves. Sketch R. If R is revolved about the y-axis, find the volume of the solid of revolution (a) by the disk/washer method, and (b) by the shell method. Show that the results are the same.

3. \( y = x^2 \)
   \( x = y^2 \)

4. \( y = 4 - x^2 \)
   \( y = 1 \)

Hints about set-up:

1. a) \( V = \pi \int_0^2 \left[ (2x)^2 - (x^2)^2 \right] dx \)  
   b) \( V = 2\pi \int_0^1 y \left[ (y) - \left( \frac{1}{2} \right) \right] dy \)

2. a) \( V = \pi \int_1^2 \left[ \left( \sqrt{x} \right)^2 - (x^3)^2 \right] dx \)  
   b) \( V = 2\pi \int_0^1 y \left[ (\sqrt{y}) - (y^2) \right] dy \)

3. a) \( V = \pi \int_0^1 \left[ \left( \sqrt{y} \right)^2 - (y^2)^2 \right] dy \)  
   b) \( V = 2\pi \int_0^1 x \left[ (\sqrt{x}) - (x^2) \right] dx \)

4. a) \( V = \pi \int_0^4 \left[ \left( \sqrt{4 - y} \right)^2 \right] dy \)  
   b) \( V = 2\pi \int_0^3 x \left[ (4 - x^2) - (1) \right] dx \)

Answers:

1. \( \frac{64\pi}{15} \)
2. \( \frac{5\pi}{14} \)

3. \( \frac{3\pi}{10} \)
4. \( \frac{9\pi}{2} \)